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"Global Stein Theorems and Hankel operators in the upper half plane"

We first give a global version on $\mathbb{R}^n$ of a well-known theorem of E. Stein, which says that the maximal function of an integrable non negative function f that is supported in a ball B is in $L^1(B)$ if and only if it belongs to the space $L\mathrm{Log}L$. Namely, a function $f \in L^1(\mathbb{R}^n)$ is in the Hardy space $H^1(\mathbb{R}^n)$ when it satisfies the two conditions :

$$\int_{\mathbb{R}^n} f dx = 0, \qquad \int_{\mathbb{R}^n} |f|(\ln_+(|f|) + \ln_+(|x|))dx < \infty,$$

and the converse holds for functions which are non negative outside a compact set and bounded below. We then give analogous conditions on integrable functions f in the upper half plane in order that their Bergman projection $P_B f$ be also integrable. We then consider $L^p - L^1$ inequalities with loss of Hankel operators.