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"Different numerical approaches for the Magnetization Inverse Problem"

This work is concerned with inverse potential problems with source term in divergence form. That is, an $\mathbb{R}^3$-valued vector field on $\mathbb{R}^3$ has to be recovered knowing (one component of) the field of the Newton potential of its divergence on a piece of surface, away from the support. Such issues typically arise in source identification from field measurements for Maxwell's equations, in the quasi-static regime. They occur for instance in geomagnetism and paleomagnetism, as well as in several non-destructive testing problems ; e.g., see [5, 8, 6] and their bibliographies. A model problem of our particular interest is inverse scanning magnetic microscopy, as considered for instance in [2, 7, 1] to recover magnetization distributions of thin rock samples, but the considerations below are of a more general and abstract nature.

The theoretical inverse problem is ill-posed since the forward operator has a kernel so extra assumptions are needed to ensure uniqueness of solutions. We will start by exploring the theoretical limitation of the inverse problem given by this kernel [3, 4]. Then, we will focus on the planar case, where we have found two cases where we could theoretically recover the original magnetization ; if we either assume that the solution is sparse (in a measure theoretical way specified in that paper but that include the standard notion of sparsity, such as a collection of separated points), or if we assume that we know a priory regions of the sample that are magnetized in a single direction. This is done by taking a measure theoretical equivalent to the group LASSO regularization technique and taking the regularizing parameter to zero. Unfortunately, this method relies in zero noise, which is not the case when working with real data and we will show what the problems of the naive use of the group LASSO technique. Then, we will show different techniques to overcome this issues, including extension of the data or changing the how the measurements are taken, together with different machine leaning techniques.

Références

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