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"Inverse Poisson Problems in divergence form. Issues of Sparsity and of Consistency."

Based on Joint work with : D. Hardin, C. Villalobos-Guillén, as well as J. Leblond, M. Nemaire.

We consider inverse Poisson problems of the form $\Delta u = \mathrm{div} M$, $u(\infty) = 0$, where a $\mathbf{R}^3$-valued source M supported on a set S is to be recovered from knowledge of ∇u away from $\mathrm{supp}\, M$. Such problems arise naturally in static Electromagnetism, like inverse magnetization problem or Electo-Encephalography.

We discuss notions of sparsity, in this infinite-dimensional context, that ensure consistency of minimizing a regularized criterion like $\|f - AM\|^2_{L^2(Q)} + \lambda \|M\|_{TV}$ where $A(M) = \nabla u$ is the forward operator, Q the measurement place, λ a regularization parameter and $\|M\|_{TV}$ is the total variation norm of M, represented as a $\mathbf{R}^3$-valued measure. Such notions depend on the kernel of A, that depends in turn of the geometry of S : the cases of open and closed surfaces turn out to be different. We also touch on some aspects of discretization, and discuss the case of volumic sample which is quite open.