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"Sharp Invertibility in Quotient Algebras of H^∞"

Given an inner function $\Theta \in H^\infty(\mathbb{D})$ and $[g]$ in the quotient algebra $H^\infty/\Theta H^\infty$, its quotient norm is $\|[g]\| := \inf\{\|g+\Theta h\|_\infty, h \in H^\infty\}$. We show that when g is normalized so that $\|[g]\| = 1$, the quotient norm of its inverse can be made arbitrarily close to 1 by imposing $|g(z)| \geq 1-\delta$ when $\Theta(z) = 0$ (the only points where one can define unambiguous values for the class $[g]$) if and only if the function Θ satisfies the following property :

$$\lim_{t\to 1} \inf\left\{|\Theta(z)| : z \in \mathbb{D}, \rho(z, \Theta^{-1}\{0\}) \geq t\right\} = 1,$$

where ρ is the usual pseudohyperbolic distance in the disc, $\rho(z,w) := \left|\frac{z-w}{1-z\bar{w}}\right|$. This last property may be satisfied or not by an inner function.

When Θ is a Blaschke product, under a condition of "super-separation" of the zeros, this property is equivalent to Θ being a thin Blaschke product.

We show that there exists Blaschke products which are interpolating and fail this property, while some Blaschke products with this property may fail to be interpolating (and thus aren't thin). We exhibit some sufficient conditions, and interesting examples.